

# *The anomaly triangle and muon $g - 2$*

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# 2-loop EW contribution to $g - 2$

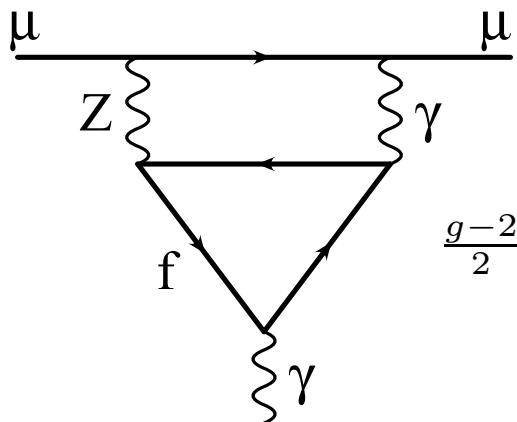
Kukhto et al. '92

SP, Perrottet, de Rafael '95

Czarnecki, Krause, Marciano '95, '96

Knecht, SP, Perrottet, de Rafael '02

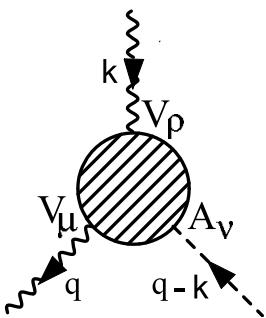
Czarnecki, Marciano, Vainshtein '03



$$\frac{g-2}{2} \propto \frac{\alpha}{\pi} \frac{G_\mu}{8\pi^2} \frac{m_\mu^2}{\sqrt{2}} \int_{m_\mu^2}^\infty dQ^2 \underbrace{\left[ w_L(Q^2) + \frac{M_Z^2}{M_Z^2 + Q^2} w_T(Q^2) \right]}_{w_L=2 \quad w_T=2 \frac{N_c}{Q^2} \quad (\text{one-loop, } m_f=0)}$$

$$\frac{g-2}{2}|_{e,u,d} \sim 2 \times 10^{-11}$$

$Q^2 = -q^2$ , "Gluon-irreducible" triangle



$$W_{\mu\nu\rho}(q, k) = T_f^{(3)} Q_f^2 \left[ w_L(Q^2) q_\nu \epsilon_{\mu\rho\alpha\beta} q^\alpha k^\beta + w_T(Q^2) k^\sigma \left( q^2 \epsilon_{\mu\nu\rho\sigma} + q_\nu \epsilon_{\mu\rho\lambda\sigma} q^\lambda + q_\mu \epsilon_{\rho\nu\lambda\sigma} q^\lambda \right) \right] + \mathcal{O}(k^2)$$

- $w_L(Q^2)$  related to the chiral anomaly  $\implies \sum_{\nu,e,u,d} w_L = 0$ .
- $w_T(Q^2)$  not but...

# Theorem

Vainshtein '02

Knecht, SP, Perrottet, de Rafael, '03

In the massless limit, to all orders in  $\alpha_s$ :

$$w_L(Q^2) = 2 w_T(Q^2)$$

and, since anomaly does not get renormalized:  $w_L = 2 \frac{N_c}{Q^2}$  ( Adler,Bardeen '69; Witten '83)

$\Rightarrow$  neither does  $2w_T = 2 \frac{N_c}{Q^2}$  , to all orders in  $\alpha_s$ .

Using  $L_\mu^{(3)} = \sum_{\ell=\nu,e} \overline{\ell_L} \gamma_\mu T^{(3)} \ell_L + \sum_{q=u,d} \overline{q_L} \gamma_\mu T^{(3)} q_L$ , etc...in  $SU(2)_L \times U(1)_Y$ :

$$Q^2 [w_L(Q^2) - 2w_T(Q^2)] \propto \int d^4x d^4y e^{iqx} (y-x)_\lambda \epsilon^{\mu\nu\rho\lambda} \underbrace{\left\langle T \left\{ \textcolor{red}{L}_\mu^{(3)}(x) \textcolor{blue}{V}_\nu^{(Y)}(y) \textcolor{red}{R}_\rho^{(Y)}(0) \right\} \right\rangle}_{\not\equiv \mathbb{1}_L \otimes \mathbb{1}_R}$$

- i.e.,  $w_L - 2w_T$  has no pert. contributions in  $\alpha_s$ , it is like, e.g.,  $\langle LR \rangle = \langle VV - AA \rangle$ .

# Non-perturbative effects

1) Adler-Bardeen-Witten :  $w_L(Q^2) = 2 \frac{N_c}{Q^2}$  (exact for all  $Q$  !)

2) However, for  $w_T(Q^2)$ :

- Large  $Q^2$ :

$$2w_T(Q^2) \approx 2 \frac{N_c}{Q^2} (1 + \textcolor{red}{NO } \alpha_s) + (\text{const.}) \alpha_s \textcolor{green}{\chi} \frac{\langle \bar{\psi} \psi \rangle^2}{Q^6} + \mathcal{O}(1/Q^8)$$

Magnetic susceptibility,  $\textcolor{green}{\chi} = \frac{\Pi_{VT}(0)}{\langle \bar{\psi} \psi \rangle}$ , very poorly known.

- Small  $Q^2$ :

$$2w_T(Q^2) \approx (\text{const.}) C_{22}^{(p^6)} + \mathcal{O}(Q^2) , \quad C_{22}^{(p^6)} \sim 1/M_{Hadron}^2 \quad (\text{unknown})$$

Chiral Pert. Theory,  $\mathcal{L}_{eff}$  (parity-odd):

$$\mathcal{L}_{\mathcal{O}(p^6)} = C_{22}^{(p^6)} \epsilon_{\mu\nu\alpha\beta} \underbrace{\text{Tr} \left( u^\mu \left\{ \nabla_\gamma f_+^{\gamma\nu}, f_+^{\alpha\beta} \right\} \right)}_{\pi, \eta, \dots} + \dots \quad (SU_{N_F} \times SU_{N_F} \rightarrow SU_{N_F})$$

# Conjectures

- Conjecture 1:  $w_L(Q^2) - 2 w_T(Q^2) = -2 \frac{N_c}{f_\pi^2} \Pi_{LR}(Q^2)$  (Son-Yamamoto '10)

in wide class of “AdS/QCD” models (chiral limit,  $N_c \rightarrow \infty$ )  
(not without caveats, e.g. OPE is exponential; mismatch in pert. theory if treated as a WI)  
( Knecht, SP, de Rafael '11)

Chiral log's respect this relation. (Gorsky, Kopnin,Krikun, Vainshtein '12)

Test:  $C_{22}^{(p^6)}(\mu) = -\frac{N_c}{32\pi^2 f_\pi^2} L_{10}^{(p^4)}(\mu)$  ??

If true at one  $\mu \Rightarrow$  true at all  $\mu$

- Conjecture 2:  $\chi = -\frac{N_c}{4\pi^2 f_\pi^2} \sim -9 \text{ GeV}^{-2}$  (Magnetic susceptibility) ??  
(Vainshtein '02):

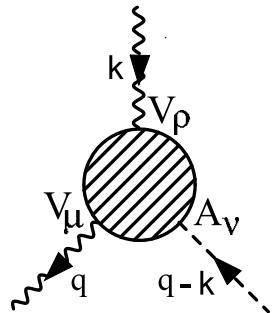
Other results:  $\chi \sim -3 \text{ GeV}^{-2}$ , sum rules, VMD, (Ioffe, Fadin, Lipatov '10; Balitsky et al. '85; Belyaev et al. '84; Ball et al. '02)

# Another Perturbative Surprise

Up to now, special kinematic configuration in  $\langle VVA \rangle$ .

Jegerlehner,Tarasov '06

However, it has been found at two loops for all momenta that :



$$W_{\mu\nu\rho}(q, k) = W_{\mu\nu\rho}(q, k)|_{\text{one-loop}} \left(1 + \underbrace{\mathcal{O}(\alpha_s)}_{=0 !!}\right)$$

i.e., no renormalization, not just for the anomaly, but for the whole triangle !

Given the non-trivial momentum dependence,

can this be just a coincidence ?

could this be true to all orders in  $\alpha_s$  ?

# Summary

- VVA is a very interesting theoretical laboratory for QCD
- Most results obtained in chiral limit: can lattice help/check ?
- The LbL  $\longleftrightarrow \langle VVA \rangle$  connection:

( Melnikov, Vainshtein '04; Prades, de Rafael, Vainshtein '09)

$$k_1 \approx k_2 \gg k_3$$

